

# Tunneling Dynamics of Bose-Einstein Condensates with Feshbach Resonances

Bambi Hu<sup>1,2</sup> and Le-Man Kuang<sup>1,3</sup>

<sup>1</sup>*Department of Physics and Centre for Nonlinear Studies,  
Hong Kong Baptist University, Hong Kong, China*

<sup>2</sup>*Department of Physics, University of Houston, Houston, Texas 77204*

<sup>3</sup>*Department of Physics, Hunan Normal University, Changsha 410081, China*

We study tunneling dynamics of atomic pairs in Bose-Einstein condensates with Feshbach resonances. It is shown that the tunneling of the atomic pairs depends on not only the tunneling coupling between the atomic condensate and the molecular condensate, but also the inter-atomic nonlinear interactions and the initial number of atoms in these condensates. It is found that in addition to oscillating tunneling current between the atomic condensate and the molecular condensate, the nonlinear atomic-pair tunneling dynamics sustains a self-locked population imbalance: macroscopic quantum self-trapping effect. Influence of decoherence induced by non-condensate atoms on tunneling dynamics is investigated. It is shown that decoherence suppresses atomic-pair tunneling.

PACS numbers: 03.75.Fi, 74.50.+r, 05.30.Jp, 32.80.Pj

The atomic Bose-Einstein condensates [1–3] offer new opportunities for studying quantum-degenerate fluids. All the essential properties of atomic Bose-Einstein condensed systems are determined by the strength of the atomic interactions. In contrast with the situation of the traditional superfluids, the strength of the inter-particle interactions in the atomic Bose-Einstein condensate can vary over a wide range of values through changing external fields. Hence one can manipulate and control condensate properties by varying the strength of interactions. The Feshbach resonance approach [4] is considered as an effective one to alter the inter-atomic interactions in Bose-Einstein condensates. The magnetic-field-induced Feshbach resonances in an atomic Bose condensate have already been observed experimentally [5]. Theoretical studies of the ultracold atoms with Feshbach resonances [6,7] showed that the interactions responsible for the Feshbach resonances produce a second condensate component, a molecular condensate, and predict that tunneling of atomic pairs occurs between the atomic condensate and the molecular condensate. Recently, the molecular Bose-Einstein condensate has been produced experimentally [8].

The purpose of this paper is to study tunneling dynamics of atomic pairs between the atomic condensate and the molecular condensate. We show that in addition to oscillating tunneling current between the atomic condensate and the molecular condensate, the nonlinearity of the tunneling dynamics sustains a self-maintained population imbalance: macroscopic quantum self-trapping effect (MQST). We also discuss the influence of decoherence induced by non-condensate atoms on tunneling dynamics and find that decoherence suppresses atomic-pair tunneling.

The binary atom Feshbach resonances are hyperfine-induced spin-flip processes that bring the colliding atoms to a bound molecular state of different spins, and then return an unbound state. These processes can be described by Hamiltonian ( $\hbar = 1$ )

$$\hat{H}_{FR} = \alpha \int d\mathbf{r} \hat{\psi}_m^+(\mathbf{r}) \hat{\psi}_a(\mathbf{r}) \hat{\psi}_a(\mathbf{r}) + h.c., \quad (1)$$

where  $\hat{\psi}_m(\mathbf{r})$ ,  $\hat{\psi}_m^+(\mathbf{r})$  ( $\hat{\psi}_a(\mathbf{r})$ ,  $\hat{\psi}_a^+(\mathbf{r})$ ) are the annihilation and creation field operators of the molecules (atoms),  $\alpha$  stands for the coupling constant. The Hamiltonian  $\hat{H}_{FR}$  together with atomic, molecular, and atom-molecule interaction Hamiltonians ( $\hbar = 1$ )

$$\begin{aligned} \hat{H}_a = & \int d\mathbf{r} \hat{\psi}_a^+(\mathbf{r}) \left[ -\frac{1}{2M} \nabla^2 + V(\mathbf{r}) \right] \hat{\psi}_a(\mathbf{r}) \\ & + \frac{\lambda'_a}{2} \int d\mathbf{r} \hat{\psi}_a^+(\mathbf{r}) \hat{\psi}_a^+(\mathbf{r}) \hat{\psi}_a(\mathbf{r}) \hat{\psi}_a(\mathbf{r}), \end{aligned} \quad (2)$$

$$\begin{aligned} \hat{H}_m = & \int d\mathbf{r} \hat{\psi}_m^+(\mathbf{r}) \left[ -\frac{1}{4M} \nabla^2 + V(\mathbf{r}) + \epsilon \right] \hat{\psi}_m(\mathbf{r}) \\ & + \frac{\lambda'_m}{2} \int d\mathbf{r} \hat{\psi}_m^+(\mathbf{r}) \hat{\psi}_m^+(\mathbf{r}) \hat{\psi}_m(\mathbf{r}) \hat{\psi}_m(\mathbf{r}), \end{aligned} \quad (3)$$

$$\hat{H}_{am} = \lambda' \int d\mathbf{r} \hat{\psi}_a^+(\mathbf{r}) \hat{\psi}_m^+(\mathbf{r}) \hat{\psi}_a(\mathbf{r}) \hat{\psi}_m(\mathbf{r}), \quad (4)$$

forms a total Hamiltonian

$$\hat{H} = \hat{H}_a + \hat{H}_m + \hat{H}_{am} + \hat{H}_{FR}, \quad (5)$$

which governs the dynamics of the system under our consideration. Here  $V(\mathbf{r})$  represents the trapped potential,  $\lambda'_{a(m)} = 4\pi a_{a(m)} / ((2)M)$  with  $M$  being the atomic mass and  $a_{a(m)}$  the scattering length,  $\lambda'$  denotes the coupling constant of atom-molecule interaction. The detuning  $\epsilon$  linearly depends on the magnetic field  $\epsilon \propto B - B_0$  with  $B_0$  being the resonant magnetic field.

For small atomic and molecular condensates [9], the atomic and molecular field operators can be approximated as  $\hat{\psi}_a(\mathbf{r}) = \hat{a}\phi_a(\mathbf{r})$ ,  $\hat{\psi}_m(\mathbf{r}) = \hat{b}\phi_b(\mathbf{r})$  where  $\phi_a(\mathbf{r})$  and  $\phi_b(\mathbf{r})$  are real normalized mode functions for the two condensates, and  $\hat{a}$  and  $\hat{b}$  are associated mode annihilation operators which satisfy the standard bosonic commutation relations. Then the total Hamiltonian becomes the two-mode Hamiltonian

$$\begin{aligned}\hat{H} &= \hat{H}_0 + \hat{H}', \\ \hat{H}_0 &= \omega_a \hat{a}^\dagger \hat{a} + \omega_b \hat{b}^\dagger \hat{b} + \lambda_a \hat{a}^{\dagger 2} \hat{a}^2 + \lambda_b \hat{b}^{\dagger 2} \hat{b}^2 + \lambda \hat{a}^\dagger \hat{a} \hat{b}^\dagger \hat{b}, \\ \hat{H}' &= \alpha (\hat{b}^\dagger \hat{a}^2 + \hat{b} \hat{a}^{\dagger 2}),\end{aligned}\quad (6)$$

where  $\alpha \hat{b}^\dagger \hat{a}^2$  describes the annihilation of a atomic pair in the atomic condensate and the creation of one molecule in the molecular condensate thereby transferring a pair of atoms from the atomic condensate to the molecular condensate with  $\alpha$  being the corresponding tunneling coupling constant. The Hermitian conjugate part  $\alpha \hat{b} \hat{a}^{\dagger 2}$  describes the reverse process. Hence, what the Feshbach Hamiltonian  $\hat{H}'$  describes is not only a three-body recombination process of the molecular formation from a pair of atoms but also a tunneling process between the atomic condensate and the molecular condensate.

In general, the Hamiltonian (6) can not be exactly solved, but it can be perturbatively solved in the off-resonant regime with a large detuning  $\epsilon \gg 0$ , by which we mean here that  $\epsilon$  greatly exceeds the Feshbach-resonant interaction energy, so we can treat  $H'$  as a perturbation. Let  $\hat{H}_0|n, m\rangle = E_{n,m}^{(0)}|n, m\rangle$  where  $|n, m\rangle$  is an eigenstate of the number operators  $\hat{a}^\dagger \hat{a}$  and  $\hat{b}^\dagger \hat{b}$  defined by  $\hat{a}^\dagger \hat{a}|n, m\rangle = n|n, m\rangle$  and  $\hat{b}^\dagger \hat{b}|n, m\rangle = m|n, m\rangle$ . It is easy to find that

$$\begin{aligned}E_{n,m}^{(0)} &= n\omega_a + m\omega_b + (n^2 - n)\lambda_a \\ &\quad + (m^2 - m)\lambda_b + nm\lambda.\end{aligned}\quad (7)$$

For simplicity, we consider the nondegenerate case and assume that  $\hat{H}|\psi_{n,m}\rangle = E_{n,m}|\psi_{n,m}\rangle$ . Then the perturbative energy and eigenstate are found to be

$$\begin{aligned}E_{n,m} &\approx E_{n,m}^{(0)} + \alpha [a_{n,m} \sqrt{n(n-1)(m+1)} \\ &\quad + b_{n,m} \sqrt{(n+1)(n+2)m}],\end{aligned}\quad (8)$$

$$\begin{aligned}|\psi_{n,m}\rangle &\approx A_{n,m} [|n, m\rangle + a_{n,m} |n-2, m+1\rangle \\ &\quad + b_{n,m} |n+2, m-1\rangle],\end{aligned}\quad (9)$$

where the normalization constant  $A_{n,m}$  and two coefficients  $a_{n,m}$  and  $b_{n,m}$  are given by

$$\begin{aligned}A_{n,m} &= \frac{1}{\sqrt{1 + a_{n,m}^2 + b_{n,m}^2}}, \\ a_{n,m} &= \frac{\alpha \sqrt{n(n-1)(m+1)}}{E_{n,m}^{(0)} - E_{n-2,m+1}^{(0)}}, \\ b_{n,m} &= \frac{\alpha \sqrt{(n+1)(n+2)m}}{E_{n,m}^{(0)} - E_{n+2,m-1}^{(0)}},\end{aligned}\quad (10)$$

Let the two condensates be initially in a state  $|\Psi(0)\rangle = \sum_{n,m} C_{n,m}(0) |\psi_{n,m}\rangle$ , the time evolution of the wave function is then given by the expression

$$|\Psi(t)\rangle \approx \sum_{n,m} C_{n,m}(0) e^{-iE_{n,m}t} |\psi_{n,m}\rangle. \quad (11)$$

In order to investigate tunneling dynamics, we introduce the population difference

$$P(t) = n_a(t) - 2n_b(t), \quad (12)$$

where  $n_{a(b)}(t)$  is the number of atoms (molecules) in the atomic (molecular) condensate at time  $t$ .

Now let us assume that initially both the atomic and molecular condensates are in a Fock state  $|n_1, n_2\rangle$ . From Eqs. (11) and (12) we find that

$$P(t) = P_0(n_1, n_2) + \sum_{i=1}^3 P_i(n_1, n_2) \cos[\omega_i(n_1, n_2)t], \quad (13)$$

where the oscillation frequencies are given by

$$\begin{aligned}\omega_1(n_1, n_2) &= E_{n_1, n_2} - E_{n_1+2, n_2-1}, \\ \omega_2(n_1, n_2) &= E_{n_1, n_2} - E_{n_1-2, n_2+1}, \\ \omega_3(n_1, n_2) &= E_{n_1+2, n_2-1} - E_{n_1-2, n_2+1},\end{aligned}\quad (14)$$

and the coefficients  $P_i(n_1, n_2)$  are defined by

$$\begin{aligned}P_0(n_1, n_2) &= A_{n_1, n_2}^2 [(n_1 - 2n_2) + (n_1 - 2n_2 - 4)a_{n_1, n_2}^2 \\ &\quad + (n_1 - 2n_2 + 4)b_{n_1, n_2}^2] \\ &\quad + A_{n_1+2, n_2-1}^2 a_{n_1+2, n_2-1}^2 [(n_1 - 2n_2 + 4) \\ &\quad + (n_1 - 2n_2)a_{n_1+2, n_2-1}^2 \\ &\quad + (n_1 - 2n_2 + 8)b_{n_1+2, n_2-1}^2] \\ &\quad + A_{n_1-2, n_2+1}^2 b_{n_1-2, n_2+1}^2 [(n_1 - 2n_2 - 4) \\ &\quad + (n_1 - 2n_2 - 6)a_{n_1-2, n_2+1}^2 \\ &\quad + (n_1 - 2n_2)b_{n_1-2, n_2+1}^2],\end{aligned}\quad (15)$$

$$\begin{aligned}P_1(n_1, n_2) &= 2A_{n_1, n_2} A_{n_1+2, n_2-1} a_{n_1+2, n_2-1} \\ &\quad \times [(n_1 - 2n_2)a_{n_1+2, n_2-1} \\ &\quad + (n_1 - 2n_2 + 4)b_{n_1, n_2}],\end{aligned}$$

$$\begin{aligned}P_2(n_1, n_2) &= 2A_{n_1, n_2} A_{n_1-2, n_2+1} b_{n_1-2, n_2+1} \\ &\quad \times [(n_1 - 2n_2 - 4)a_{n_1, n_2} \\ &\quad + (n_1 - 2n_2)b_{n_1-2, n_2+1}],\end{aligned}$$

$$\begin{aligned}P_3(n_1, n_2) &= 2A_{n_1+2, n_2-1} A_{n_1-2, n_2+1} a_{n_1+2, n_2-1}^2 \\ &\quad \times b_{n_1-2, n_2+1}^2 (n_1 - 2n_2).\end{aligned}\quad (16)$$

From Eq. (12) we see that the population difference between the atomic condensate and the molecular condensate exhibits oscillating behaviors. Especially, we can obtain the nonzero time average of the population difference labeled by  $\bar{P}$  given by

$$\bar{P} = P_0(n_1, n_2), \quad (17)$$

which implies that there is a self-locked population imbalance between the atomic condensate and the molecular condensate. This is the MQST which occurs in the usual two Bose condensate system [10,11] as well. It is easy to check that the MQST vanishes when the nonlinearities in interactions are absent. Hence the MQST phenomenon is a nonlinear effect.

Eq. (12) indicates that the population imbalance between the atomic condensate and the molecular condensate exhibits nonlinear oscillations with the time evolution. It is these oscillations that leads to a Josephson-like tunneling current between the atomic condensate and molecular condensate, which can be defined as  $I(t) = \dot{P}(t)/N$  with  $N$  being the total number of atoms in the atomic and molecular condensates. Making use of Eq. (13) it is easy to find that

$$I(t) = - \sum_{i=1}^3 \frac{P_i(n_1, n_2) \omega_i(n_1, n_2)}{n_1 + 2n_2} \sin[\omega_i(n_1, n_2)t]. \quad (18)$$

From Eqs. (7)-(16) and (18) we see that the tunneling of the atomic pairs depends on not only the tunneling coupling between the atomic condensate and the molecular condensate but also the inter-atomic nonlinear interactions and the initial number of atoms in these condensates.

In order to further understand the influence of inter-atomic interactions on tunneling dynamics of the system under our consideration, let us specialize to the case of the atomic condensate initially being in a number state  $|N\rangle$ , and the molecular condensate initially being unpopulated, i.e.,  $|\Psi(0)\rangle = |N, 0\rangle$ . In this case, the population difference and the tunneling current between the two condensates given by the expressions

$$P(t) = P_0(N, 0) + P_2(N, 0) \cos[\omega_2(N, 0)t], \quad (19)$$

$$I(t) = -I_{am} \sin[\omega_2(N, 0)t], \quad (20)$$

where we have set  $I_{am} = P_2(N, 0)\omega_2(N, 0)/N$ .

It is easy to see that the amplitude  $I_{am}$  and the frequency  $\omega_2(N, 0)$  depend upon the initial number of atoms in the condensates, the tunneling coupling  $\alpha$ , the nonlinear interaction strengths  $\lambda_a$ ,  $\lambda_b$ , and  $\lambda$ . To find out how the initial number of atoms in the condensates and the interaction strengths affect the tunneling current, in Figure 1 we plot the amplitude of the tunneling current as a function of interaction strengths for different initial number of atoms when the initial state is  $|\Psi(0)\rangle = |N, 0\rangle$ ,  $\lambda_a = \lambda_b = \lambda$ , and  $\omega_a = 2\omega_b$ . Figure 1 indicates that the amplitude of the tunneling current is almost independent of the initial number of atoms, the tunneling coupling, and inter-atomic nonlinear interactions in the regime of weak (strong) tunneling coupling (nonlinear couplings)  $0 < \alpha/\lambda < 4$ . However, the tunneling coupling and the inter-atomic nonlinear interactions strongly affect the amplitude of the tunneling current in the regime of strong (weak) tunneling coupling (nonlinear couplings)  $\alpha/\lambda > 4$ . From Figure 1 we can see that the amplitude of the tunneling current increases with increasing both the initial number of atoms and the tunneling coupling in the regime of strong (weak) tunneling coupling (nonlinear couplings)  $\alpha/\lambda > 4$ .

In Figure 2, we display the scaled frequency of the tunneling current,  $\omega/\lambda = \omega_2(N, 0)/\lambda$ , as a function of interaction strengths for different initial number of atoms

when the initial state is  $|\Psi(0)\rangle = |N, 0\rangle$ ,  $\lambda_a = \lambda_b = \lambda$ , and  $\omega_a = 2\omega_b$ . It is interesting to note that from Figure 2 we can see that there exists a zero-frequency point, labeled by D. From Eqs. (14) it is straight forward to see that the zero-frequency point is a degenerate point of energy of the system under our consideration, at which the nondegenerate perturbation theory is broken. From Eqs. (7), (8), (14), and (20) we can find that the degenerate point is given by the expression  $\alpha/\lambda = [(3N - 4)(7N - 20)/2(N - 2)(N - 3)]^{1/2}$ . Figure 2 indicates that on the left hand side of the degenerate point D the scaled frequency decreases with increasing the tunneling coupling and/or decreasing the nonlinear interaction strengths, and increases with increasing the initial number of atoms. On the other hand, on the left hand side of the degenerate point D the scaled frequency increases with increasing both the tunneling coupling and the initial number of atoms, and/or decreasing the nonlinear interaction strengths.

However, it is customary to consider a Bose-Einstein condensate to be in a coherent state, associated with a macroscopic wave function with both amplitude and a phase, the presence of which is originated from Bose broken symmetry. Assume that the two condensate are initially in the coherent states  $|\alpha\rangle$  and  $|\beta\rangle$ , which are eigenstates of  $\hat{a}$  and  $\hat{b}$ , respectively, then we have

$$C_{n,m}(0) = \frac{A_{n,m}}{\exp(|\alpha|^2 + |\beta|^2)} \left[ \frac{\alpha^n \beta^m}{\sqrt{n!m!}} + \frac{a_{n,m} \alpha^{n-2} \beta^{m+1}}{\sqrt{(n-2)!(m+1)!}} + \frac{b_{n,m} \alpha^{n+2} \beta^{m-1}}{\sqrt{(n+2)!(m-1)!}} \right]. \quad (21)$$

Making use of Eqs. (11), (12) and (21) we find the expression of the population difference

$$P(t) = 2 \sum_{n,m} \left\{ \frac{1}{2} p_0(n, m) C(n, m; n, m) + p_1(n, m) C(n, m; n-2, m+1) \times \cos(E_{n,m} - E_{n-2, m+1})t + p_2(n, m) C(n, m; n+2, m-1) \times \cos(E_{n,m} - E_{n+2, m-1})t + p_3(n, m) C(n, m; n+4, m-2) \times \cos(E_{n,m} - E_{n+4, m-2})t \right\}, \quad (22)$$

where we have introduced the notations

$$p_0(n, m) = (n-2m) + (n-2m-4)a_{n,m}^2 + (n-2m+4)b_{n,m}^2, \quad (23)$$

$$p_1(n, m) = (n-2m-4)a_{n,m},$$

$$p_2(n, m) = (n-2m+4)b_{n,m},$$

$$p_3(n, m) = (n-2m+4)a_{n+4, m-2}b_{n,m}, \quad (24)$$

and

$$C(n, m; n', m') = C_{n, m}(0)C_{n', m'}^*(0). \quad (25)$$

From Eq. (22) we can get the nonzero time average of the population difference labeled by  $\bar{P}$  given by

$$\bar{P} = \sum_{n, m} C(n, m; n, m) p_0(n, m), \quad (26)$$

which implies that there exists the MQST between the atomic condensate and the molecular condensate.

From Eq. (22) we can obtain the Josephson-like tunneling current

$$\begin{aligned} I(t) = & - \sum_{n, m} [I_1(n, m) \sin(E_{n, m} - E_{n-2, m+1})t \\ & - I_2(n, m) \sin(E_{n, m} - E_{n+2, m-1})t \\ & - I_3(n, m) \sin(E_{n, m} - E_{n+4, m-2})t], \end{aligned} \quad (27)$$

with

$$\begin{aligned} I_1(n, m) = & 2p_1(n, m)C(n, m; n-2, m+1) \\ & \times (E_{n, m} - E_{n-2, m+1})/N, \\ I_2(n, m) = & 2p_2(n, m)C(n, m; n+2, m-1) \\ & \times (E_{n, m} - E_{n+2, m-1})/N, \\ I_3(n, m) = & 2p_3(n, m)C(n, m; n+4, m-2) \\ & \times (E_{n, m} - E_{n+4, m-2})/N. \end{aligned} \quad (28)$$

where  $N = |\alpha|^2 + 2|\beta|^2$  is the total number of the atoms in the two condensates.

We now discuss the effect of the decoherence. In experiments on trapped Bose condensates of atomic gases, condensate atoms continuously interact with non-condensate atoms (environment). As is well known, interactions between a quantum system and environment cause two types of unwelcomed effects: dissipation and decoherence [12]. The dissipation effect, which dissipates the energy of the quantum system into the environment, is characterized by the relaxation time scale  $\tau_r$ . In contrast, the decoherence effect is much more insidious because the coherence information leaks out into the environment in another time scale  $\tau_d$ , which is much shorter than  $\tau_r$ . Since macroscopic quantum phenomena in Bose-Einstein condensates mainly depend on  $\tau_d$  rather than  $\tau_r$ , the discussions in present paper only focus on the decoherence problem rather than the dissipation effect.

We use a reservoir consisting of an infinite set of harmonic oscillators to model environment of condensate atoms and molecules in a trap, and assume the total Hamiltonian [13] to be

$$\begin{aligned} \hat{H}_T = & \hat{H} + \sum_k \omega_k \hat{b}_k^\dagger \hat{b}_k + F(\{\hat{S}\}) \sum_k c_k (\hat{b}_k^\dagger + \hat{b}_k) \\ & + F(\{\hat{S}\})^2 \sum_k \frac{c_k^2}{\omega_k^2}, \end{aligned} \quad (29)$$

where the second term is the Hamiltonian of the reservoir. The last term is a renormalization term. The third term represents the interaction between the system and the reservoir with a coupling constant  $c_k$ , where  $\{\hat{S}\}$  is a set of linear operators of the system or their linear combinations in the same picture as that of  $\hat{H}$ ,  $F(\{\hat{S}\})$  is an operator function of  $\{\hat{S}\}$ . In order to enable what the interaction between the system and environment describes is decoherence not dissipation, we require that the linear operator  $\hat{S}$  commutes with the the Hamiltonian of the system  $\hat{H}$ . Then, the interaction term commutes with the Hamiltonian of the system. This implies that there is no energy transfer between the system and its environment. So that it does describe the decoherence. The concrete form of the function  $F(\{\hat{S}\})$ , which may be considered as an experimentally determined quantity, may be different for different environment.

The Hamiltonian  $\hat{H}_T$  can be exactly solved by using the unitary transformation  $\hat{U} = \exp[\hat{H} \sum_k (c_k/\omega_k)(\hat{b}_k^\dagger - \hat{b}_k)]$ . Corresponding to the Hamiltonian (29), the total density operator of the system plus reservoir can be expressed as  $\hat{\rho}_T(t) = e^{-i\hat{H}t} \hat{U}^{-1} e^{-it \sum_k \omega_k \hat{b}_k^\dagger \hat{b}_k} \hat{\rho}_T(0) \hat{U} e^{it \sum_k \omega_k \hat{b}_k^\dagger \hat{b}_k}$ . We assume that the system and reservoir are initially in thermal equilibrium and uncorrelated, so that  $\hat{\rho}_T(0) = \hat{\rho}(0) \otimes \hat{\rho}_R$ , where  $\hat{\rho}(0)$  is the initial density operator of the system, and  $\hat{\rho}_R$  the density operator of the reservoir, which can be written as  $\hat{\rho}_R = \prod_k \hat{\rho}_k(0)$  with  $\hat{\rho}_k(0)$  is the density operator of the  $k$ -th harmonic oscillator in thermal equilibrium. After taking the trace over the reservoir, we can get the reduced density operator of the system, denoted by  $\hat{\rho}(t) = \text{tr}_R \hat{\rho}_T(t)$ , whose matrix elements in the eigenstate representation of  $\hat{H}$  are explicitly written as

$$\begin{aligned} \rho_{(m', n')(m, n)}(t) = & |\rho_{(m', n')(m, n)}(0)| e^{-\gamma_{(m', n')(m, n)}(t)} \\ & \times e^{-i\phi_{(m', n')(m, n)}(t)}, \end{aligned} \quad (30)$$

where the damping factor and the phase shift are defined by

$$\gamma_{(m', n')(m, n)}(t) = v_-^2(m', n'; m, n) Q_2(t), \quad (31)$$

$$\begin{aligned} \phi_{(m', n')(m, n)}(t) = & v_+(m', n'; m, n) v_-(m', n'; m, n) Q_1(t) \\ & + \theta_{(m', n')(m, n)}, \end{aligned} \quad (32)$$

where we have introduced the following notations:

$$v_\pm(n, m; n', m') = F(\{S(n, m)\}) \pm F(\{S(n', m')\}), \quad (33)$$

$$\rho_{(m', n')(m, n)}(0) = |\rho_{(m', n')(m, n)}(0)| e^{-i\theta_{(m', n')(m, n)}}, \quad (34)$$

and the two reservoir-dependent functions are given by

$$Q_1(t) = \int_0^\infty d\omega J(\omega) \frac{c^2(\omega)}{\omega^2} \sin(\omega t), \quad (35)$$

$$Q_2(t) = 2 \int_0^\infty d\omega J(\omega) \frac{c^2(\omega)}{\omega^2} \sin^2\left(\frac{\omega t}{2}\right) \coth\left(\frac{\beta\omega}{2}\right). \quad (36)$$

Here we have taken the continuum limit of the reservoir modes:  $\sum_k \rightarrow \int_0^\infty d\omega J(\omega)$ , where  $J(\omega)$  is the spectral density of the reservoir,  $c(\omega)$  is the continuum expression for  $c_k$ , and  $\beta = 1/k_B T$  with  $k_B$  and  $T$  being the Boltzmann constant and temperature, respectively.

Eq. (30) indicates that the interaction between the system and its environment induces a phase shift and a decaying factor in the reduced density operator of the system. We now consider the population difference between the atomic condensate and the molecular condensate in the presence of the decoherence defined by  $P(t) = \text{Tr} \hat{\rho}(t)(\hat{n}_a - 2\hat{n}_b)$ . Making use of Eq. (30), We find that

$$P(t) = \frac{1}{2} \sum_{n,m} \{ p_0(n,m) |\rho_{(n,m)(n,m)}(0)| \cos \theta_{(n,m)(n,m)} + p_1(n,m) |\rho_{(n,m)(n-2,m+1)}(0)| \times \cos \phi_{(n,m)(n-2,m+1)}(t) e^{-\gamma_{(n,m)(n-2,m+1)}(t)} + p_2(n,m) |\rho_{(n,m)(n+2,m-1)}(0)| \times \cos \phi_{(n,m)(n+2,m-1)}(t) e^{-\gamma_{(n,m)(n+2,m-1)}(t)} + p_3(n,m) |\rho_{(n,m)(n+4,m-2)}(0)| \times \cos \phi_{(n,m)(n+4,m-2)}(t) e^{-\gamma_{(n,m)(n+4,m-2)}(t)} \}. \quad (37)$$

Then, the tunneling current is given by

$$I(t) = - \sum_{n,m} \{ p_1(n,m) |\rho_{(n,m)(n-2,m+1)}(0)| \times [\dot{\gamma}_{(n,m)(n-2,m+1)}(t) + \dot{\phi}_{(n,m)(n-2,m+1)}(t) \times \sin \phi_{(n,m)(n-2,m+1)}(t)] e^{-\gamma_{(n,m)(n-2,m+1)}(t)} + p_2(n,m) |\rho_{(n,m)(n+2,m-1)}(0)| \times [\dot{\gamma}_{(n,m)(n+2,m-1)}(t) + \dot{\phi}_{(n,m)(n+2,m-1)}(t) \times \sin \phi_{(n,m)(n+2,m-1)}(t)] e^{-\gamma_{(n,m)(n+2,m-1)}(t)} + p_3(n,m) |\rho_{(n,m)(n+4,m-2)}(0)| \times [\dot{\gamma}_{(n,m)(n+4,m-2)}(t) + \dot{\phi}_{(n,m)(n+4,m-2)}(t) \times \sin \phi_{(n,m)(n+4,m-2)}(t)] e^{-\gamma_{(n,m)(n+4,m-2)}(t)} \}. \quad (38)$$

From Eqs. (37) and (38) we can immediately draw one important qualitative conclusion: since  $\gamma_{(n,m)(n',m')}$  is positive definite, the existence of the decoherence is always to tend to suppress the population difference and tunneling current between the atomic condensate and the molecular condensate.

From Eqs. (31), (32), and (35)-(38) we see that all necessary information about the effects of the environment on the population difference and the tunneling current is contained in the spectral density of the reservoir. To proceed further let us now specialize to the Ohmic case with the spectral distribution  $J(\omega) = [\eta\omega/c^2(\omega)] \exp(-\omega/\omega_c)$ , where  $\omega_c$  is the high frequency cut-off,  $\eta$  is a positive characteristic parameter of the reservoir. With this choice, at low temperature the functions  $Q_1(t)$  and  $Q_2(t)$  are given by the expressions  $Q_1(t) = \eta \tan^{-1}(\omega_c t)$  and  $Q_2(t) = \eta \{ \frac{1}{2} \ln[1 +$

$(\omega_c t)^2] + \ln[\frac{\beta}{\pi t} \sinh(\frac{\pi t}{\beta})] \}$ . In particular, At zero temperature and in the meaningful domain of time  $\omega_c t \gg 1$ , we have  $Q_1(t) \doteq \eta/(\omega_c t^2)$ , and  $Q_2(t) \doteq \eta \ln(\omega_c t)$ . Then we find

$$P(t) = \frac{1}{2} \sum_{n,m} \{ p_0(n,m) |\rho_{(n,m)(n,m)}(0)| \cos \theta_{(n,m)(n,m)} + p_1(n,m) |\rho_{(n,m)(n-2,m+1)}(0)| \times \cos \phi_{(n,m)(n-2,m+1)}(t) (\omega_c t)^{-(\eta_{1nm}^-)^2} + p_2(n,m) |\rho_{(n,m)(n+2,m-1)}(0)| \times \cos \phi_{(n,m)(n+2,m-1)}(t) (\omega_c t)^{-(\eta_{2nm}^-)^2} + p_3(n,m) |\rho_{(n,m)(n+4,m-2)}(0)| \times \cos \phi_{(n,m)(n+4,m-2)}(t) (\omega_c t)^{-(\eta_{3nm}^-)^2} \}, \quad (39)$$

$$I(t) = - \sum_{n,m} \{ (p_1(n,m) |\rho_{(n,m)(n-2,m+1)}(0)| [\frac{(\eta_{1nm}^-)^2}{t} + \frac{\eta_{1nm}^{+-}}{\omega_c t^2} \sin \phi_{(n,m)(n-2,m+1)}(t)] (\omega_c t)^{-(\eta_{1nm}^-)^2} + p_2(n,m) |\rho_{(n,m)(n+2,m-1)}(0)| [(\eta_{2nm}^-)^2 + \frac{\eta_{2nm}^{+-}}{\omega_c} \sin \phi_{(n,m)(n+2,m-1)}(t)] (\omega_c t)^{-(\eta_{2nm}^-)^2} + p_3(n,m) |\rho_{(n,m)(n+4,m-2)}(0)| [(\eta_{3nm}^-)^2 + \frac{\eta_{3nm}^{+-}}{\omega_c} \sin \phi_{(n,m)(n+4,m-2)}(t)] (\omega_c t)^{-(\eta_{3nm}^-)^2} \}, \quad (40)$$

where we have used  $\eta_{1nm}^\pm = \sqrt{\eta} v_\pm(n, m; n-2, m+1)$ ,  $\eta_{2nm}^\pm = \sqrt{\eta} v_\pm(n, m; n+2, m-1)$ ,  $\eta_{3nm}^\pm = \sqrt{\eta} v_\pm(n, m; n+4, m-2)$ , and  $\eta_{inm}^{+-} = \eta_{inm}^+ \eta_{inm}^-$ . Eqs. (39) and (40) indicates that the tunneling current decays away according to the ‘‘power law’’, where we have noted that the decay factors can not be taken outside the summation on the r.h.s. of Eqs. (39) and (40).

In summary, we have studied tunneling dynamics of atomic pairs in atomic Bose-Einstein condensates with Feshbach resonances, and shown that the tunneling of the atomic pairs depends on not only the tunneling coupling between the atomic condensate and the molecular condensate, but also the inter-atomic nonlinear interactions and the initial number of atoms in these condensates. especially, we have shown that the tunneling coupling and the inter-atomic nonlinear interactions strongly affect the tunneling of atomic pairs in the regime of strong (weak) tunneling coupling (nonlinear couplings) when the atomic condensate is in a number state and the molecular condensate in the vacuum state. This implies that the tunneling of atomic pairs between the atomic condensate and the molecular condensate can be manipulated and controlled by varying the tunneling coupling and/or inter-atomic nonlinear interaction strengths. We have revealed the existence of the MQST between the atomic condensate and the molecular condensate. The MQST is a kind of nonlinear effects which vanishes in the

absence of the inter-atomic nonlinear interactions. We have also discussed the influence of decoherence induced by non-condensate atoms on the tunneling dynamics, and shown that decoherence suppresses the atomic-pair tunneling. Finally, it should be mentioned that inelastic collisions [6,14] between the atomic and molecular condensates and reservoir may affect the atomic-pair tunneling between the atomic and molecular condensates. Influence of inelastic collisions between the atomic and molecular condensates can be taken account into through introducing an imaginary part in the interaction strengths  $\lambda$  and  $\lambda_b$  [6]. Inelastic collisions between the system of the atomic-molecular condensates and reservoir lead to dissipation. A detailed investigation on the dissipation problem of the system of the atomic-molecular condensates is beyond the scope of the present paper, and will be given elsewhere.

### ACKNOWLEDGMENTS

This work was supported in part by grants from Hong Kong Research Grants Council (RGC) and the Hong Kong Baptist University Faculty Research Grant (FRG). L.M.K. also acknowledges support from the Climbing Project of China and NSF of China, the Excellent Young-Teacher Foundation of the Educational Commission of China, ECF and STF of Hunan Province. The authors would like to thank Dr. J.H. Xiao for his useful discussions and help in preparing figures of the paper.

- 
- [1] M.H. Anderson, J.R. Ensher, M.R. Matthews, C.E. Wieman, and E.A. Cornell, *Science* 269, 198 (1995).
  - [2] K.B. Davis, M.-O. Mewes, M.R. Andrews, N.J. van Druten, D.S. Durfee, D.M. Kurn, and W. Ketterle, *Phys. Rev. Lett.* 75, 3969 (1995).
  - [3] C.C. Bradley, C.A. Sackett, and R.G. Hulet, *Phys. Rev. Lett.* 75, 1687 (1995); 78, 985 (1997).
  - [4] A. J. Moerdijk, B. J. Verhaar, and A. Axelsson, *Phys. Rev. A* 51, 4852 (1995); J. M. Vogels, C.C. Tsai, R.S. Freeland, S.J.J.M.F. Kokkelmans, B.J. Verhaar, and D.J. Heinzen, *Phys. Rev. A* 56, R1067 (1997); F.H. Mies, E. Tiesinga, and P.S. Julienne, *Phys. Rev. A* 61, 022721 (2000).
  - [5] S. Inouye, M.R. Andrews, J. Stenger, H.-J. Miesner, D.M. Stamper-Kurn, and W. Ketterle, *Nature* 392, 151 (1998); Ph. Courteille, R.S. Freeland, D.J. Heinzen, F.A. van Abeelen, and B.J. Verhaar, *Phys. Rev. Lett.* 81, 69 (1998); V. Vuletić, A.J. Kerman, C. Chin, and S. Chu, *Phys. Rev. Lett.* 82, 1406 (1999).
  - [6] E. Timmermans, P. Tommasini, R. Côte, M. Hussein, and A. Kerman, *Phys. Rev. Lett.* 83, 2691 (1999); E. Timmermans, P. Tommasini, M. Hussein, and A. Kerman, *Phys. Rep.* 315, 199 (1999); *cond-mat/9805323* (1998); P. Tommasini, E. Timmermans, M. Hussein, and A. Kerman, *cond-mat/9804015* (1998).

- [7] F. A. van Abeelen and B. J. Verhaar, *Phys. Rev. Lett.* 83, 1550 (1999).
- [8] R. Wynar, R.S. Freeland, D.J. Han, C. Ryu, and D.J. Heinzen, *Science* 287, 1016 (2000).
- [9] D. Gordon and C. M. Savage, *Phys. Rev. A* 59, 4623 (1999); M. J. Steel and M. J. Collett, *Phys. Rev. A* 57, 2920 (1998); G. J. Milburn, J. Corney, E.M. Wright, D.F. Walls, *Phys. Rev. A* 55, 4318 (1997); E. M. Wright and D. F. Walls, *Phys. Rev.* 77, 2158 (1996); A. Imamoglu, M. Lewenstein, and L. You, *Phys. Rev. Lett.* 78, 2511 (1997).
- [10] A. Smerzi, S. Giovanazzi, and S. R. Shenoy, *Phys. Rev. Lett.* 79, 4950 (1997); S. Raghavan, A. Smerzi, S. Fantoni, and S. R. Shenoy, *Phys. Rev. A* 59, 620 (1999).
- [11] L.-M Kuang and Z.-W. Ouyang, *Phys. Rev. A* 61, 023604 (2000).
- [12] W. H. Zurek, *Phys. Today* 44 (10), 36 (1991); I. L. Chuang, R. Laflamme, P.W. Shor, and W.H. Zurek, *Science* 270, 1633 (1991); C. P. Sun, H. Zhan, and X.F. Liu, *Phys. Rev. A* 58, 1810 (1998); L.-M Kuang and X. Chen, *Phys. Rev. A* 56, 3139 (1997).
- [13] L.-M Kuang, H.-S. Zeng, and Z.-Y. Tong, *Phys. Rev. A* 60, 3815 (1999); L.-M Kuang, Z.-Y. Tong, Z.-W. Ouyang, and H.-S. Zeng, *Phys. Rev. A* 61, 013608 (2000).
- [14] J. Stenger, M.R. Andrews, H.-J. Miesner, D.M. Stamper-Kurn, and W. Ketterle, *Phys. Rev. Lett.* 82 (1999) 2422; V.A. Yurovsky, A. Ben-Reuven, P.S. Julienne, and C.J. Williams, *Phys. Rev. A* 60, R765 (1999).

### Figure Captions

FIG. 1. Amplitude of the tunneling current as a function of interaction strengths for different initial number of atoms when the initial state is  $|\Psi(0)\rangle = |N, 0\rangle$ ,  $\lambda_a = \lambda_b = \lambda$ , and  $\omega_a = 2\omega_b$ . Here we have set  $\omega = \omega_2(N, 0)$ .

FIG. 2. Scaled frequency of the tunneling current as a function of interaction strengths for different initial number of atoms when the initial state is  $|\Psi(0)\rangle = |N, 0\rangle$ ,  $\lambda_a = \lambda_b = \lambda$ , and  $\omega_a = 2\omega_b$ . Here we have set  $\omega = \omega_2(N, 0)$ .

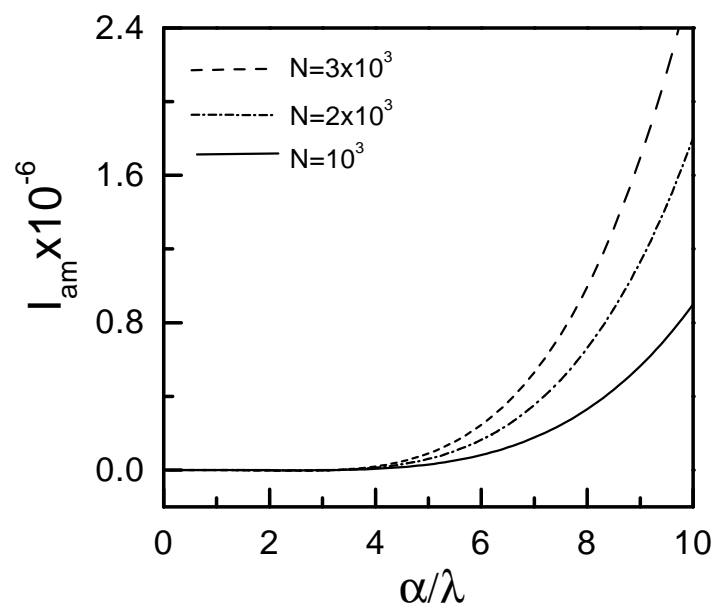


Fig.1

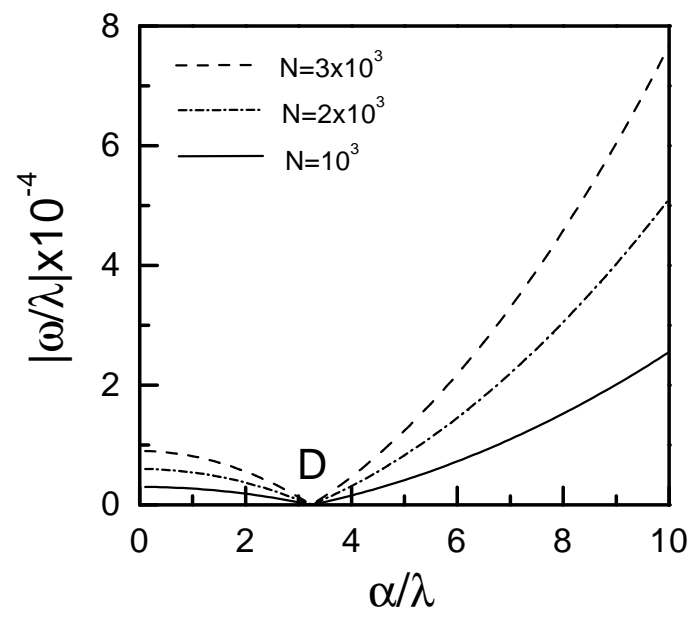


Fig.2